

**Electricity and New Energy**

# **Single-Phase AC Power Circuits**

**Course Sample**

579368

Order no.: 579368 (Printed version) 591850 (CD-ROM)

First Edition

Revision level: 06/2018

By the staff of Festo Didactic

© Festo Didactic Ltée/Ltd, Quebec, Canada 2010

Internet: [www.festo-didactic.com](http://www.festo-didactic.com)

e-mail: [did@de.festo.com](mailto:did@de.festo.com)

Printed in Canada

All rights reserved

ISBN 978-2-89640-400-1 (Printed version)

ISBN 978-2-89640-711-8 (CD-ROM)

Legal Deposit – Bibliothèque et Archives nationales du Québec, 2010

Legal Deposit – Library and Archives Canada, 2010

The purchaser shall receive a single right of use which is non-exclusive, non-time-limited and limited geographically to use at the purchaser's site/location as follows.

The purchaser shall be entitled to use the work to train his/her staff at the purchaser's site/location and shall also be entitled to use parts of the copyright material as the basis for the production of his/her own training documentation for the training of his/her staff at the purchaser's site/location with acknowledgement of source and to make copies for this purpose. In the case of schools/technical colleges, training centers, and universities, the right of use shall also include use by school and college students and trainees at the purchaser's site/location for teaching purposes.

The right of use shall in all cases exclude the right to publish the copyright material or to make this available for use on intranet, Internet, and LMS platforms and databases such as Moodle, which allow access by a wide variety of users, including those outside of the purchaser's site/location.

Entitlement to other rights relating to reproductions, copies, adaptations, translations, microfilming, and transfer to and storage and processing in electronic systems, no matter whether in whole or in part, shall require the prior consent of Festo Didactic.

















Information in this document is subject to change without notice and does not represent a commitment on the part of Festo Didactic. The Festo materials described in this document are furnished under a license agreement or a nondisclosure agreement.

Festo Didactic recognizes product names as trademarks or registered trademarks of their respective holders.













All other trademarks are the property of their respective owners. Other trademarks and trade names may be used in this document to refer to either the entity claiming the marks and names or their products. Festo Didactic disclaims any proprietary interest in trademarks and trade names other than its own.

# Safety and Common Symbols

The following safety and common symbols may be used in this course and on the equipment:

Symbol	Description
	<b>DANGER</b> indicates a hazard with a high level of risk which, if not avoided, will result in death or serious injury.
	<b>WARNING</b> indicates a hazard with a medium level of risk which, if not avoided, could result in death or serious injury.
	<b>CAUTION</b> indicates a hazard with a low level of risk which, if not avoided, could result in minor or moderate injury.
	<b>CAUTION</b> used without the <i>Caution, risk of danger</i> sign  , indicates a hazard with a potentially hazardous situation which, if not avoided, may result in property damage.
	Caution, risk of electric shock
	Caution, hot surface
	Caution, risk of danger. Consult the relevant user documentation.
	Caution, lifting hazard
	Caution, belt drive entanglement hazard
	Caution, chain drive entanglement hazard
	Caution, gear entanglement hazard
	Caution, hand crushing hazard
	Notice, non-ionizing radiation
	Consult the relevant user documentation.
	Direct current

# Safety and Common Symbols

Symbol	Description
	Alternating current
	Both direct and alternating current
	Three-phase alternating current
	Earth (ground) terminal
	Protective conductor terminal
	Frame or chassis terminal
	Equipotentiality
	On (supply)
	Off (supply)
	Equipment protected throughout by double insulation or reinforced insulation
	In position of a bi-stable push control
	Out position of a bi-stable push control

# Table of Contents

Preface.....	XI
About This Course .....	XIII
To the Instructor .....	XV
<b>Unit 1      Alternating Current.....</b>	<b>1</b>
DISCUSSION OF FUNDAMENTALS.....	1
DC circuits versus ac circuits .....	1
Alternating current (ac) and ac voltage .....	2
Alternating current and ac voltage supplied by public power distribution utilities.....	2
Safety rules .....	3
<b>Ex. 1-1      The Sine Wave .....</b>	<b>5</b>
DISCUSSION .....	5
Relationship between a rotating phasor and a sine wave .....	5
Period and frequency of a sinusoidal voltage or current.....	8
Amplitude and instantaneous value of a sinusoidal voltage or current.....	8
Effective or root-mean-square (rms) value and heating capacity .....	10
Effective (rms) value of a sinusoidal voltage or current .....	10
PROCEDURE .....	11
Setup and connections.....	11
Measuring voltage, current, and frequency in an ac circuit ....	13
Relationship between frequency and period.....	15
Measuring voltage, current, and frequency in a series ac circuit.....	17
CONCLUSION .....	20
REVIEW QUESTIONS .....	20
<b>Ex. 1-2      Phase Angle and Phase Shift.....</b>	<b>21</b>
DISCUSSION .....	21
Phase angle .....	21
Phase shift .....	23
PROCEDURE .....	25
Setup and connections.....	25
Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit.....	27
Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit .....	34
CONCLUSION .....	42
REVIEW QUESTIONS .....	43

# Table of Contents

<b>Ex. 1-3</b>	<b>Instantaneous Power and Average Power.....</b>	<b>45</b>
	DISCUSSION .....	45
	Instantaneous power.....	45
	Average power .....	46
	Rationale behind rms values.....	47
	PROCEDURE .....	48
	Setup and connections.....	48
	Average power measurements .....	50
	Rationale behind rms values.....	54
	CONCLUSION .....	56
	REVIEW QUESTIONS.....	56
<b>Unit 2</b>	<b>Resistance, Reactance, and Impedance .....</b>	<b>63</b>
	DISCUSSION OF FUNDAMENTALS.....	63
	Introduction to inductors and capacitors .....	63
	Distinction between resistance, reactance, and impedance.....	65
<b>Ex. 2-1</b>	<b>Inductive Reactance.....</b>	<b>67</b>
	DISCUSSION .....	67
	Inductors and inductive reactance .....	67
	Inductive phase shift .....	68
	PROCEDURE .....	68
	Setup and connections.....	68
	Inductance and inductive reactance .....	70
	Effect of frequency on the inductive reactance .....	72
	Measuring the inductive phase shift.....	74
	CONCLUSION .....	77
	REVIEW QUESTIONS.....	77
<b>Ex. 2-2</b>	<b>Capacitive Reactance.....</b>	<b>79</b>
	DISCUSSION .....	79
	Capacitors and capacitive reactance .....	79
	Capacitive phase shift.....	80
	PROCEDURE .....	80
	Setup and connections.....	80
	Capacitance and capacitive reactance .....	82
	Effect of the frequency on the capacitive reactance .....	85
	Measuring the capacitive phase shift.....	86
	CONCLUSION .....	89
	REVIEW QUESTIONS.....	89

# Table of Contents

<b>Ex. 2-3</b>	<b>Impedance .....</b>	<b>91</b>
	DISCUSSION .....	91
	Phasor diagrams related to resistors, inductors, and capacitors .....	91
	Phasor diagram related to a resistor .....	92
	Phasor diagram related to an inductor .....	92
	Phasor diagram related to a capacitor .....	93
	Equivalent reactance of series-connected reactive components .....	93
	Impedance of resistors, inductors, and capacitors connected in series .....	95
	Impedance of resistors, inductors, and capacitors connected in parallel .....	98
	PROCEDURE .....	99
	Setup and connections .....	99
	Equivalent reactance of a series LC circuit .....	101
	Impedance of a series RL circuit .....	104
	Impedance of a series RC circuit .....	105
	Impedance of a series RLC circuit .....	107
	Impedance of a parallel RL circuit .....	108
	Impedance of a parallel RC circuit .....	110
	CONCLUSION .....	112
	REVIEW QUESTIONS .....	112
<b>Unit 3</b>	<b>Power in AC Circuits .....</b>	<b>115</b>
	DISCUSSION OF FUNDAMENTALS .....	115
	Introduction to active, reactive, and apparent power .....	115
<b>Ex. 3-1</b>	<b>Active and Reactive Power .....</b>	<b>117</b>
	DISCUSSION .....	117
	Active power in a resistor .....	117
	Reactive power in an inductor .....	118
	Reactive power in a capacitor .....	119
	Power Meter .....	120
	PROCEDURE .....	121
	Setup and connections .....	121
	Active power in a resistor .....	122
	Reactive power in an inductor .....	125
	Reactive power in a capacitor .....	127
	CONCLUSION .....	130
	REVIEW QUESTIONS .....	130

# Table of Contents

<b>Ex. 3-2</b>	<b>Apparent Power and the Power Triangle .....</b>	<b>133</b>
	DISCUSSION .....	133
	Phasor diagrams related to active and reactive power .....	133
	Phasor diagram related to the active power in a resistor .....	133
	Phasor diagram related to the reactive power in an inductor ...	134
	Phasor diagram related to the reactive power in a capacitor ...	134
	Apparent power .....	136
	Power triangle .....	137
	Power factor .....	137
	PROCEDURE .....	138
	Setup and connections.....	138
	Total reactive power in a circuit .....	139
	Apparent power, power factor, and power triangle .....	142
	CONCLUSION .....	146
	REVIEW QUESTIONS.....	147
<b>Unit 4</b>	<b>AC Circuits Analysis .....</b>	<b>153</b>
	DISCUSSION OF FUNDAMENTALS.....	153
	Introduction to circuit analysis .....	153
	Circuit analysis methods .....	153
<b>Ex. 4-1</b>	<b>Solving Simple AC Circuits Using Circuit Impedance</b>	
	<b>Calculation .....</b>	<b>155</b>
	DISCUSSION .....	155
	Solving simple parallel circuits .....	155
	Example .....	156
	Solving simple series circuits .....	157
	Example .....	158
	PROCEDURE .....	159
	Setup and connections.....	159
	Solving a simple parallel ac circuit.....	160
	Solving a simple series ac circuit.....	163
	CONCLUSION .....	166
	REVIEW QUESTIONS.....	166
<b>Ex. 4-2</b>	<b>Solving AC Circuits Using the Power Triangle Method.....</b>	<b>169</b>
	DISCUSSION .....	169
	Solving ac circuits using the power triangle method.....	169
	Example .....	170
	PROCEDURE .....	173
	Setup and connections.....	173
	Solving an ac circuit using the power triangle method.....	175



# Table of Contents

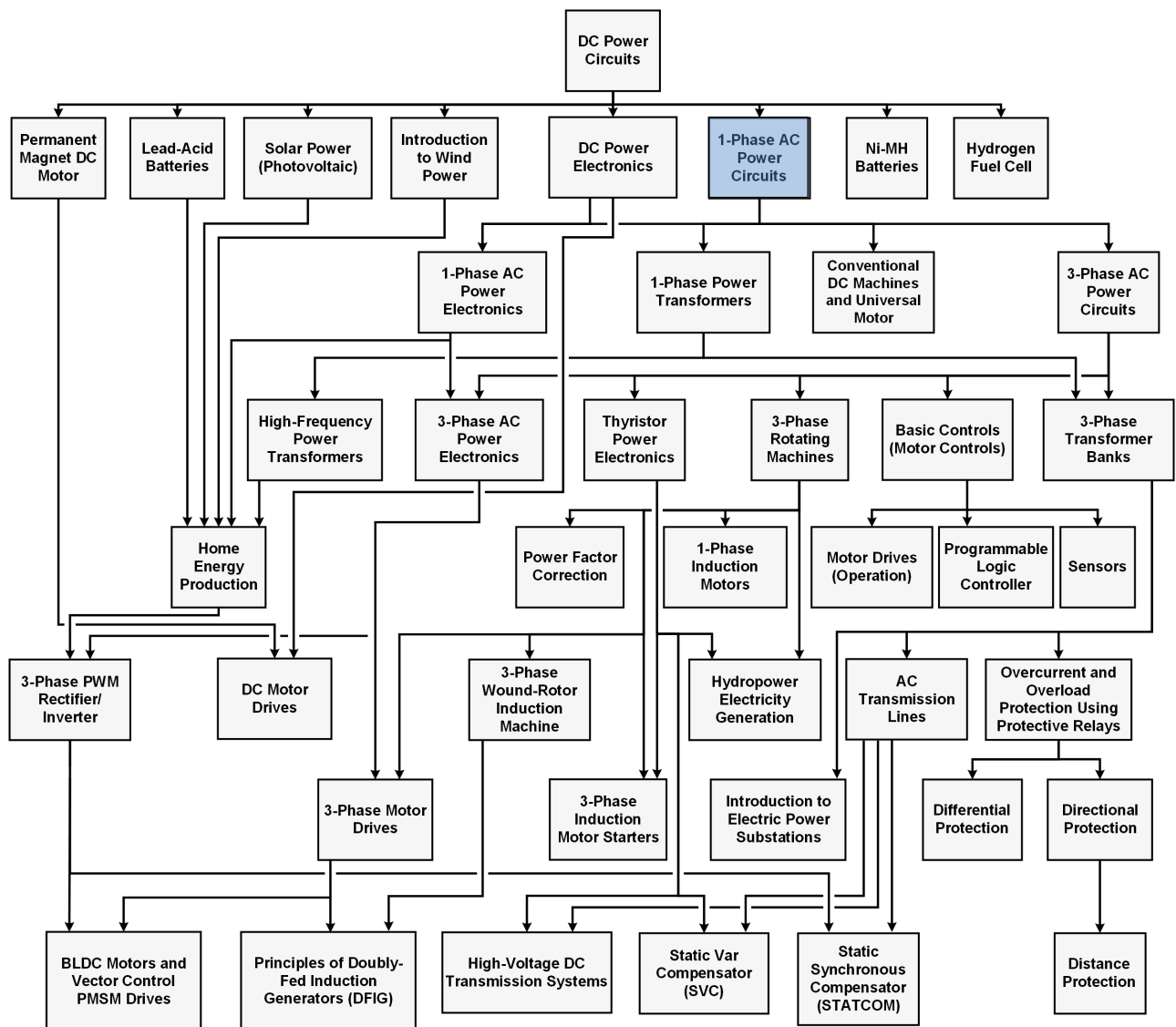
CONCLUSION .....	178
REVIEW QUESTIONS .....	179
<b>Appendix A Equipment Utilization Chart .....</b>	<b>187</b>
<b>Appendix B Glossary of New Terms.....</b>	<b>189</b>
<b>Appendix C Impedance Table for the Load Modules.....</b>	<b>193</b>
<b>Appendix D Vectorial Calculations .....</b>	<b>195</b>
Vectorial division .....	196
Vectorial multiplication .....	197
<b>Appendix E Circuit Diagram Symbols.....</b>	<b>199</b>
Index of New Terms.....	205
Bibliography .....	207



# Preface

The production of energy using renewable natural resources such as wind, sunlight, rain, tides, geothermal heat, etc., has gained much importance in recent years as it is an effective means of reducing greenhouse gas (GHG) emissions. The need for innovative technologies to make the grid smarter has recently emerged as a major trend, as the increase in electrical power demand observed worldwide makes it harder for the actual grid in many countries to keep up with demand. Furthermore, electric vehicles (from bicycles to cars) are developed and marketed with more and more success in many countries all over the world.

To answer the increasingly diversified needs for training in the wide field of electrical energy, the Electric Power Technology Training Program was developed as a modular study program for technical institutes, colleges, and universities. The program is shown below as a flow chart, with each box in the flow chart representing a course.



The Electric Power Technology Training Program.

# Preface

The program starts with a variety of courses providing in-depth coverage of basic topics related to the field of electrical energy such as ac and dc power circuits, power transformers, rotating machines, ac power transmission lines, and power electronics. The program then builds on the knowledge gained by the student through these basic courses to provide training in more advanced subjects such as motor starters and drives, storage of electrical energy in batteries, home energy production from renewable resources (wind and sunlight), large-scale electricity production from hydropower, protective relaying, and smart-grid technologies (SVC, STATCOM, HVDC transmission systems, etc.).

We invite readers to send us their tips, feedback, and suggestions for improving the course.

Please send these to [did@de.festo.com](mailto:did@de.festo.com).

The authors and Festo Didactic look forward to your comments.

# About This Course

Alternating-current (ac) power systems began to develop quickly in the late 19<sup>th</sup> century, following key developments in the field of electricity, mainly the invention of the polyphase system of electrical distribution by scientist Nikola Tesla, and the development of mathematical analysis of electricity by Charles Steinmetz, James Clerk Maxwell, and William Thomson (Lord Kelvin).

The main advantage of ac power systems is that high amounts of power can be transmitted efficiently over long transmission lines. Step-up transformers are used at the ac power generating point to increase the voltage and decrease the current. The power lost as heat in the resistance of a transmission line increases by the square of the current. Therefore, ac power is transmitted at very high voltages and low currents to reduce power losses in the line resistance to a minimum. At the receiving end of the line, step-down transformers reduce the voltage and increase the current to levels compatible with residential or industrial equipment.

Today ac power systems are used throughout the world for driving motors and powering electric equipment in transport, heating, lighting, communications, and computation.

This course, *Single-Phase AC Power Circuit*, introduces students to the fundamentals of alternating current, such as the sine wave, period and frequency, phase angle and phase shift, instantaneous and average power, etc. Students then become familiar with the inductor and capacitor. The course continues with more advanced topics such as the impedance, active power, reactive power, apparent power, and power triangle. The course concludes by teaching students how to solve ac power circuits using the impedance calculation method or the power triangle method.



**Most lighting in urban centers is powered using single-phase alternative current.**

# About This Course

## **Safety considerations**

Safety symbols that may be used in this course and on the equipment are listed in the Safety and Common Symbols table at the beginning of this document.

Safety procedures related to the tasks that you will be asked to perform are indicated in each exercise.

Make sure that you are wearing appropriate protective equipment when performing the tasks. You should never perform a task if you have any reason to think that a manipulation could be dangerous for you or your teammates.

## **Prerequisite**

As a prerequisite to this course, you should have completed course *DC Power Circuits*.

## **Systems of units**

Units are expressed using the International System of Units (SI) followed by units expressed in the U.S. customary system of units (between parentheses).

# To the Instructor

You will find in this Instructor Guide all the elements included in the Student Manual together with the answers to all questions, results of measurements, graphs, explanations, suggestions, and, in some cases, instructions to help you guide the students through their learning process. All the information that applies to you is placed between markers and appears in red.

## **Accuracy of measurements**

The numerical results of the hands-on exercises may differ from one student to another. For this reason, the results and answers given in this course should be considered as a guide. Students who correctly perform the exercises should expect to demonstrate the principles involved and make observations and measurements similar to those given as answers.

## **Equipment installation**

In order for students to be able to perform the exercises in the Student Manual, the Electric Power Technology Training Equipment must have been properly installed, according to the instructions given in the user guide Electric Power Technology Training Equipment.





Sample  
Extracted from  
Instructor Guide



## Phase Angle and Phase Shift

### EXERCISE OBJECTIVE

When you have completed this exercise, you will know what a phase angle is and how the phase angle modifies the initial displacement of a sine wave. You will be able to determine the phase shift between two sine waves, either by comparing their phase angles or by determining their separation in time. You will also know how to distinguish a leading phase shift from a lagging phase shift.

### DISCUSSION OUTLINE

The Discussion of this exercise covers the following points:

- Phase angle
- Phase shift

### DISCUSSION

#### Phase angle

As you have seen in Exercise 1-1, the graphical representation of a sine wave can be expressed by the following equation:

$$a(t) = A \sin(\omega t) \quad (1-8)$$

where  $a(t)$  is the instantaneous value of the sine wave at a given instant  $t$ .

$A$  is the amplitude of the sine wave.

$\omega$  is the angular velocity, expressed in radians per second (rad/s).

$t$  is the time, expressed in seconds (s).

This equation assumes that the sine wave cycle begins at the exact moment when  $t = 0$  (as is shown in Figure 1-10). As you will see later, this is not always the case. To represent the initial position of the sine wave, the notion of **phase angle**  $\theta$  is introduced in the equation below:

$$a(t) = A \sin(\omega t + \theta) \quad (1-9)$$

where  $\theta$  is the phase angle of the sine wave, expressed in degrees ( $^\circ$ ) or radians (rad).

From Equation (1-9), it is easy to observe that the initial value (i.e., the value at  $t = 0$ ) of the sine wave depends entirely on the phase angle  $\theta$  because the term  $\omega t$  equals 0 at  $t = 0$ . In other words, the phase angle  $\theta$  determines by how much the value of a sine wave differs from 0 at time  $t = 0$ , and thus, the position in time of the sine wave.

Figure 1-10 shows a sine wave with a phase angle  $\theta$  of  $0^\circ$ . The initial value of this sine wave is 0 because  $A \sin(\omega \cdot 0 + 0) = 0$ . This sine wave is identical to those seen in Exercise 1-1, as a phase angle value of  $0^\circ$  was implied by the absence of  $\theta$  in the equations given in Exercise 1-1.

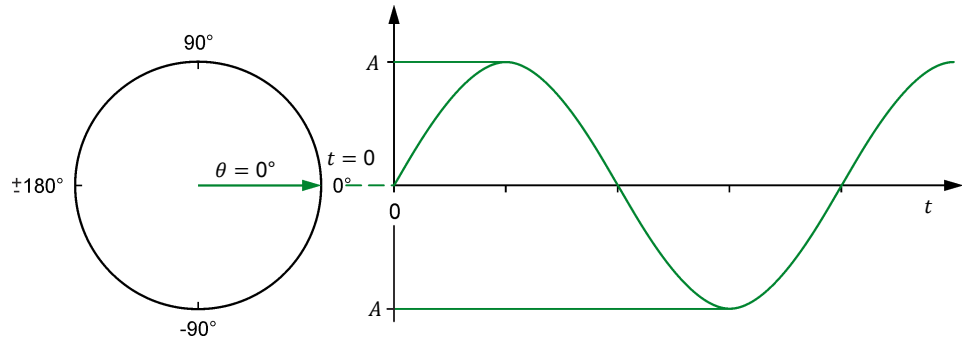


Figure 1-10. Sine wave with a phase angle  $\theta$  of  $0^\circ$ .

Figure 1-11 shows a sine wave with a phase angle  $\theta$  of  $45^\circ$ . As you can see from the figure, a positive phase angle ( $0^\circ$  to  $180^\circ$ ) results in the sine wave having a positive instantaneous value when  $t = 0$ . In other words, a positive phase angle shifts the sine wave toward the left, i.e., advances the sine wave in time.

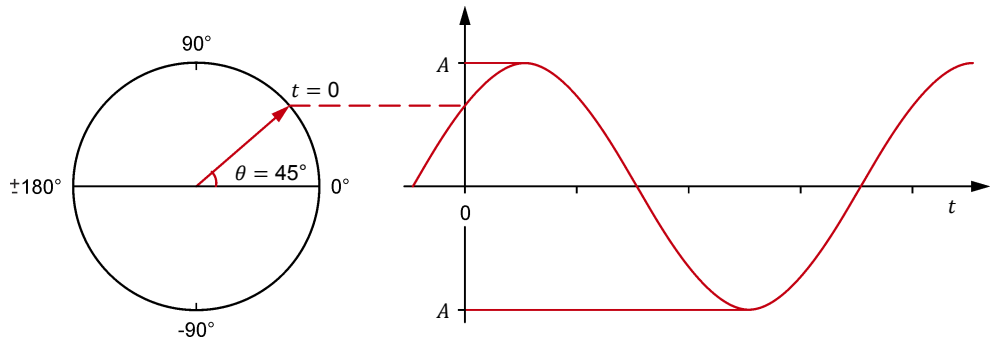


Figure 1-11. Sine wave with a phase angle  $\theta$  of  $45^\circ$ .

Figure 1-12 shows a sine wave with a phase angle  $\theta$  of  $-60^\circ$ . A negative phase angle ( $0^\circ$  to  $-180^\circ$ ) results in the sine wave having a negative instantaneous value when  $t = 0$ . In other words, a negative phase angle shifts the sine wave toward the right, i.e., delays the sine wave in time.

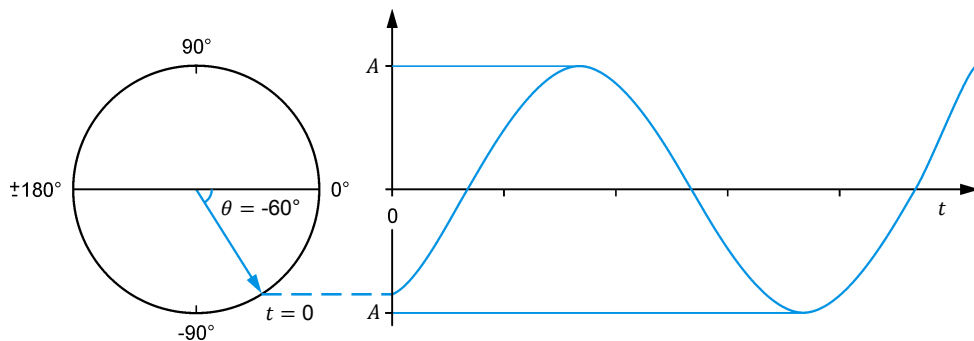


Figure 1-12. Sine wave with a phase angle  $\theta$  of  $-60^\circ$ .

Figure 1-10 to Figure 1-12 also show the phasor representations of the sine waves at time  $t = 0$ . Notice that, in each figure, the vertical distance between the tip of the rotating phasor representing the sine wave matches the instantaneous value of the sine wave at  $t = 0$ .

### Phase shift

When comparing two sine waves having the same frequency, the difference between their respective phase angles is called the **phase shift** and is expressed in degrees ( $^{\circ}$ ) or radians (rad). The magnitude of the phase shift indicates the extent of separation in time between the two sine waves, while the polarity of the phase shift (positive or negative) indicates the relationship in time between the two sine waves (**leading** or **lagging**). The sine wave amplitude value has no effect on the phase shift, as it does not change the period nor the frequency of the sine wave. Sine waves with different frequencies and, as an extension, different periods, cannot be compared by using their phase angles as their cycles do not correspond.

The phase shift between two sine waves is expressed as an angle representing a portion of a complete cycle of the sine waves. One of the two sine waves is used as the reference for phase shift measurements. The phase shift is calculated by subtracting the phase angle  $\theta_{Ref.}$  of the reference sine wave from the phase angle  $\theta$  of the sine wave of interest. This is written as an equation below.

$$\text{Phase shift} = \theta - \theta_{Ref.} \quad (1-10)$$

- where  $\theta$  is the phase angle of the sine wave of interest, expressed in degrees ( $^{\circ}$ ) or radians (rad).  
 $\theta_{Ref.}$  is the phase angle of the reference sine wave, expressed in degrees ( $^{\circ}$ ) or radians (rad).

Figure 1-13 is an example showing how the phase shift between two sine waves (X and Y) can be calculated using their phase angles.

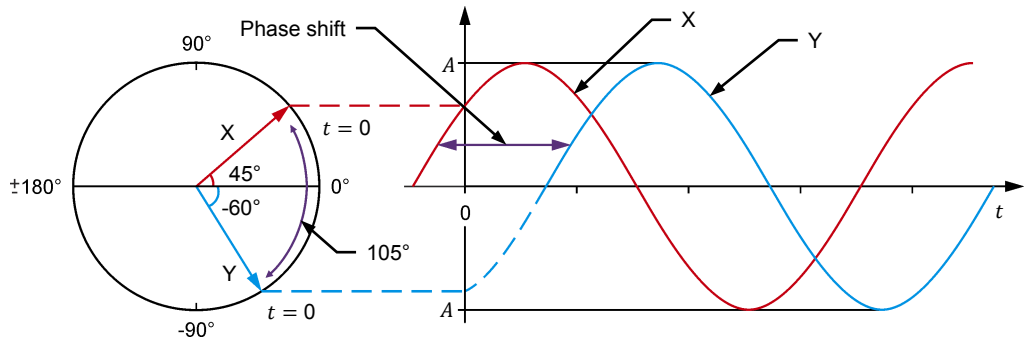


Figure 1-13. Phase shift between two sine waves with phase angles of  $45^{\circ}$  and  $-60^{\circ}$ .

In the figure, sine wave X has a phase angle  $\theta$  of  $45^{\circ}$ , while sine wave Y has a phase angle  $\theta$  of  $-60^{\circ}$ . Depending on which sine wave is used as the reference, the phase shift can be  $+105^{\circ}$  or  $-105^{\circ}$ . When sine wave X is considered as the reference, the phase shift of sine wave Y with respect to sine wave X is  $-105^{\circ}$  ( $-60^{\circ} - 45^{\circ} = -105^{\circ}$ ). The minus sign in this phase shift value indicates

that sine wave Y lags reference sine wave X. For this reason, this phase shift value can also be expressed as  $105^\circ$  lagging. Conversely, when sine wave Y is considered as the reference, the phase shift of sine wave X with respect to sine wave Y is  $+105^\circ$  ( $45^\circ - (-60^\circ) = +105^\circ$ ). The plus sign in this phase shift value indicates that sine wave X leads reference sine wave Y. For this reason, this phase shift value can also be expressed as  $105^\circ$  leading. Note that whenever two sine waves have different phase angles, the phase shift value is not zero, and thus, these sine waves are said to be out of phase.

It is possible to determine the phase shift between two sine waves of the same frequency without knowing their respective phase angles  $\theta$ . The following equation is used:

$$\text{Phase shift} = \frac{d}{T} \times 360^\circ = \frac{d}{T} \times 2\pi \text{ rad} \quad (1-11)$$

where  $d$  is the time interval between a given reference point on each of the two sine waves, expressed in seconds (s).

$T$  is the period of the sine waves, expressed in seconds (s).

This equation shows in a concrete way why it is not possible to calculate the phase shift between two sine waves having different frequencies  $f$ , as a common period  $T$  ( $T = 1/f$ ) is needed for the equation to be valid.

Consider, for example, the sine waves shown in Figure 1-14. Using Equation (1-11), the phase shift between the two sine waves is equal to:

$$\text{Phase shift} = \frac{d}{T} \times 360^\circ = \frac{3.33 \text{ ms}}{20.0 \text{ ms}} \times 360^\circ = 60^\circ$$

When sine wave 1 is used as the reference, the phase shift is lagging because sine wave 2 is delayed with respect to sine wave 1. Conversely, when sine wave 2 is considered as the reference, the phase shift is leading because sine wave 1 is in advance with respect to sine wave 2.

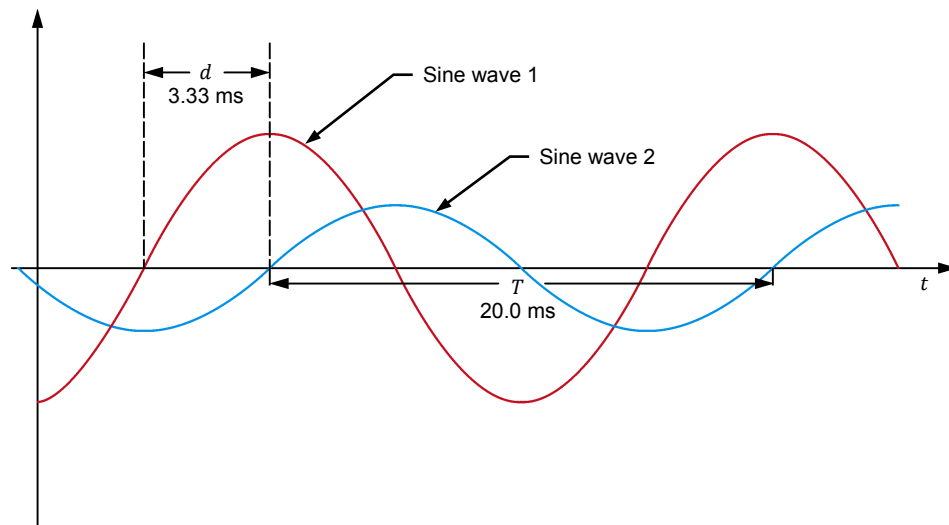


Figure 1-14. Phase shift between two sine waves having the same frequency.

## PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit
- Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit

## PROCEDURE



High voltages are present in this laboratory exercise. Do not make or modify any banana jack connections with the power on unless otherwise specified.

### Setup and connections

*In this section, you will connect an ac circuit containing an inductor and a resistor in series and set up the equipment to measure the source voltage  $E_S$ , as well as the voltage across the resistor  $E_R$ .*

1. Refer to the Equipment Utilization Chart in Appendix A to obtain the list of equipment required to perform this exercise.

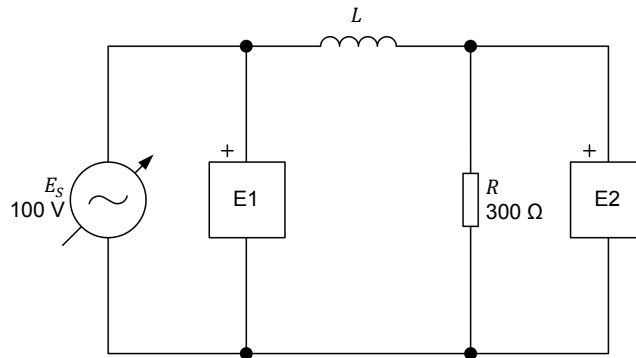
Install the required equipment in the [Workstation](#).

2. Make sure that the main power switch on the [Four-Quadrant Dynamometer/Power Supply](#) is set to the **O** (off) position, then connect its [Power Input](#) to an ac power outlet.
3. Connect the [Power Input](#) of the [Data Acquisition and Control Interface](#) to a 24 V ac power supply. Turn the 24 V ac power supply on.
4. Turn the [Four-Quadrant Dynamometer/Power Supply](#) on, then set the [Operating Mode](#) switch to [Power Supply](#). This setting allows the [Four-Quadrant Dynamometer/Power Supply](#) to operate as a power supply.
5. Connect the USB port of the [Data Acquisition and Control Interface](#) to a USB port of the host computer.

Connect the USB port of the [Four-Quadrant Dynamometer/Power Supply](#) to a USB port of the host computer.

6. Turn the host computer on, then start the [LVDAC-EMS](#) software.

7. In the **LVDAC-EMS Start-Up** window, make sure that the **Data Acquisition and Control Interface** and the **Four-Quadrant Dynamometer/Power Supply** are detected. Make sure that the **Computer-Based Instrumentation** function for the **Data Acquisition and Control Interface** is available. Select the network voltage and frequency that correspond to the voltage and frequency of your local ac power network, then click the **OK** button to close the **LVDAC-EMS Start-Up** window.
8. Set up the circuit shown in Figure 1-15. This circuit contains a resistor  $R$  and an inductor  $L$ . Inductors are studied in the next unit of this manual.



**Figure 1-15. AC circuit with a resistor and an inductor.**

The value of inductor  $L$  in the circuit of Figure 1-15 is referred to as the inductance and is expressed in henries (H). The inductance value to be used depends on the frequency of the ac power source as is indicated in Table 1-2.



As indicated in Appendix A, use the **Inductive Load** module to obtain the required inductance when the ac power network frequency is 60 Hz. Use the **Inductive and Capacitive Loads** module to obtain the required inductance when the ac power network frequency is 50 Hz.

**Table 1-2. Inductance values for 50 and 60 Hz frequencies.**

Power source frequency (Hz)	Inductance (H)
50	0.96
60	0.80

Make the necessary switch settings on the **Resistive Load** in order to obtain the resistance values required.



Appendix C of this manual lists the switch settings to implement on the **Resistive Load** in order to obtain various resistance values.

Make the necessary connections and switch settings on the **Inductive Load** (or on the **Inductive and Capacitive Loads**) in order to obtain the inductance value required.



If necessary, ask your instructor to assist you to obtain the inductance value required.



Use inputs *E1* and *E2* of the [Data Acquisition and Control Interface](#) to measure the source voltage  $E_S$  and the voltage across the resistor  $E_R$ , respectively.

9. In [LVDAC-EMS](#), open the [Four-Quadrant Dynamometer/Power Supply](#) window, then make the following settings:
  - Set the *Function* parameter to *AC Power Source*.
  - Make sure that the *Voltage Control* parameter is set to *Knob*. This allows the ac power source to be controlled manually.
  - Set the *Voltage (V at no load)* parameter to 100 V.
  - Set the *Frequency* parameter to the frequency of your local ac power network.
  - Leave the other parameters set as they are.

### Measuring the phase shift between two voltage sine waves in a resistor-inductor (RL) circuit

*In this section, you will observe the waveforms (sine waves) of the source voltage  $E_S$  and the resistor voltage  $E_R$ , using the Oscilloscope to determine the phase shift between the two sine wave voltages. Then, using the Phasor Analyzer, you will measure the phase shift between the source voltage phasor and the resistor voltage phasor and compare it to the phase shift determined from the voltage waveforms.*



*As you will see later, due to the presence of an inductor in the circuit, the circuit current lags behind the source voltage. As a result, the voltage  $E_R$  measured across the resistor is out of phase with respect to the source voltage  $E_S$ .*

10. In [LVDAC-EMS](#), open the [Metering](#) window. Set meters *E1* and *E2* to measure the rms values of the source voltage  $E_S$  and voltage  $E_R$  across the resistor  $R$ , respectively.

In the [Four-Quadrant Dynamometer/Power Supply](#) window, enable the ac power source. Readjust the value of the *Voltage (V at no load)* parameter so that the ac power source voltage  $E_S$  (indicated by meter *E1* in the [Metering](#) window) is equal to 100 V.

11. In [LVDAC-EMS](#), open the [Oscilloscope](#) and display  $E_S$  (input *E1*) and  $E_R$  (input *E2*) on channels 1 and 2, respectively. If necessary, set the time base so as to display at least two cycles of the sine waves. Place the traces of the two channels at the same vertical position.

12. Measure the period  $T$  of the source voltage  $E_S$  using the Oscilloscope then record the value below.



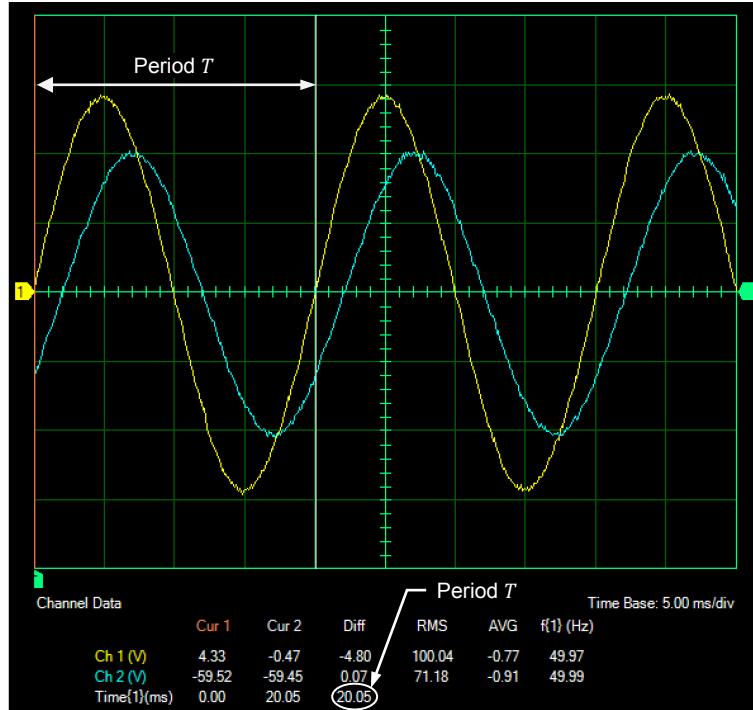
To obtain an accurate measurement, you can use the vertical cursors of the Oscilloscope to measure the period or any other time interval.

Period  $T$  = \_\_\_\_\_ ms

50 Hz: Period  $T$  = 20.05 ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising

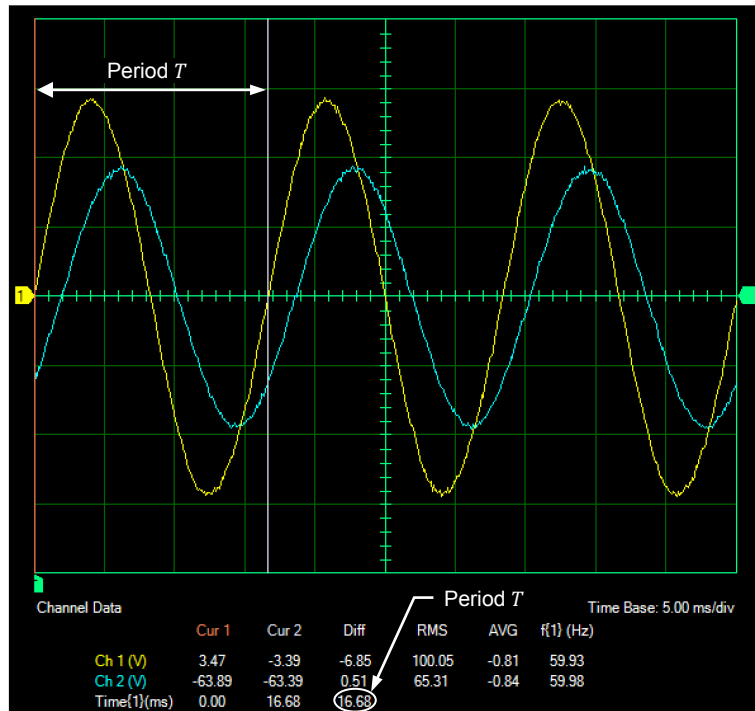


Period  $T$  of the waveform of the source voltage  $E_S$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.68$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



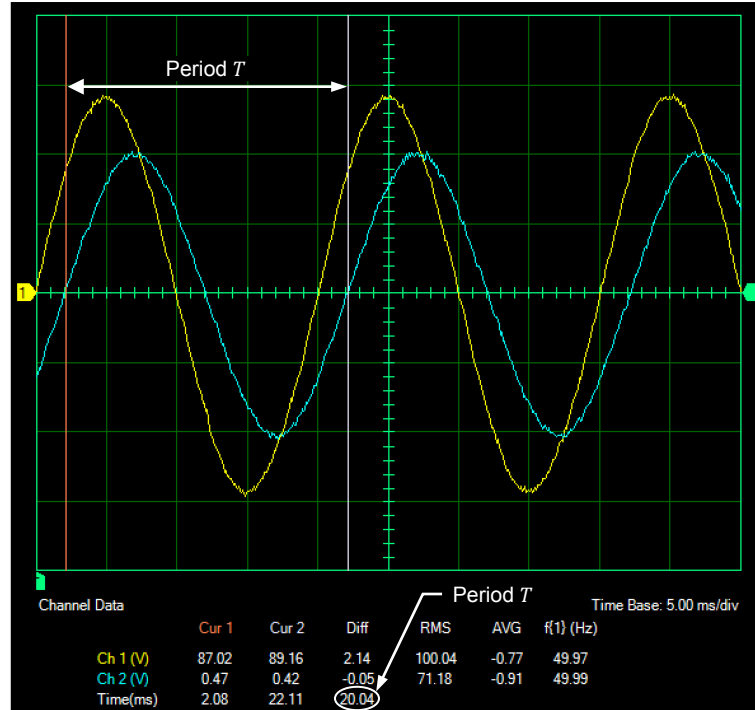
Period  $T$  of the waveform of the source voltage  $E_S$  at a frequency of 60 Hz.

13. Measure the period  $T$  of the resistor voltage  $E_R$  using the Oscilloscope then record the value below.

Period  $T =$  \_\_\_\_\_ ms

50 Hz: Period  $T = 20.04$  ms. The results are shown in the following figure.

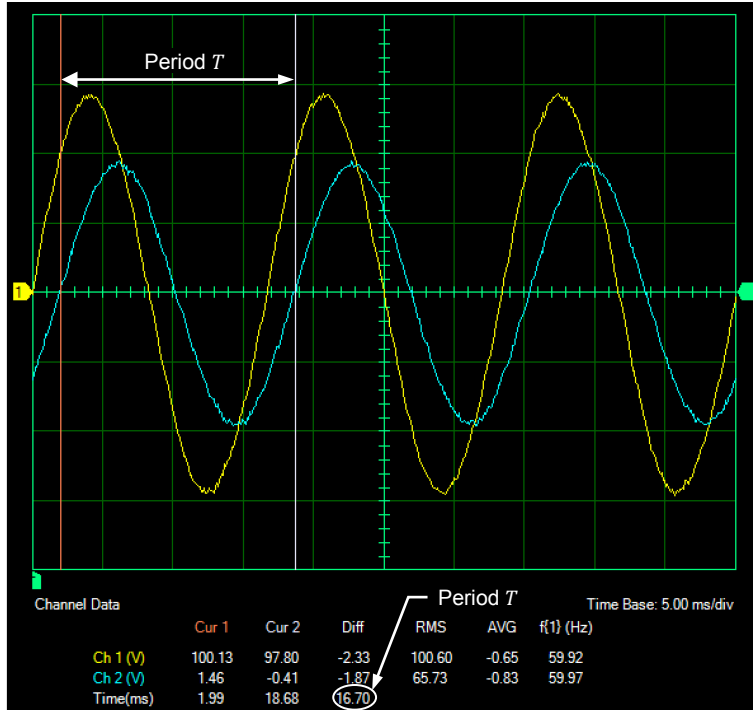
Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.70$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 60 Hz.

14. Compare the period  $T$  of the resistor voltage  $E_R$  measured in the previous step with the period  $T$  of the source voltage  $E_S$  recorded in step 12. Are the values close to each other?

Yes     No

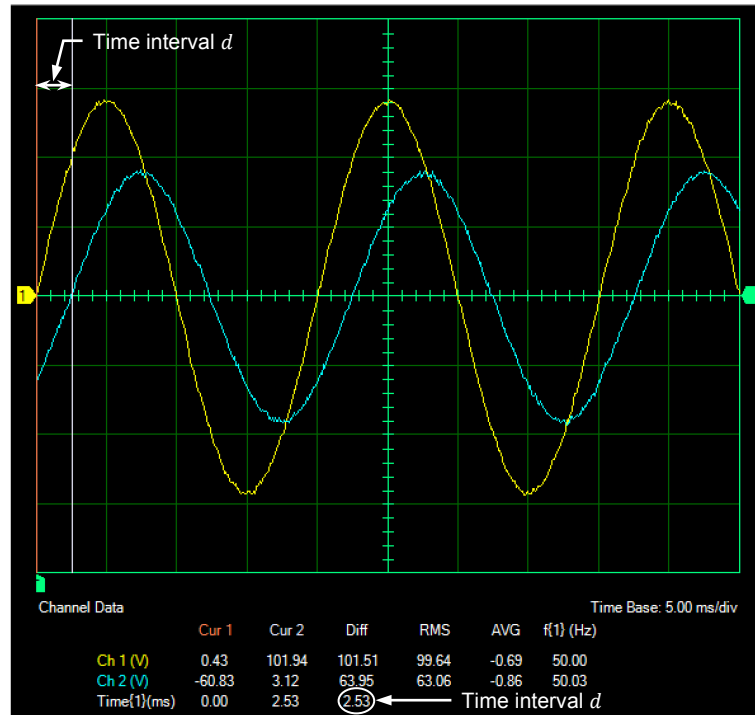
Yes

15. Measure the time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  by using the Oscilloscope.

Time interval  $d = \underline{\hspace{2cm}}$  ms

50 Hz: Time interval  $d = 2.53$  ms. The results are shown in the following figure.

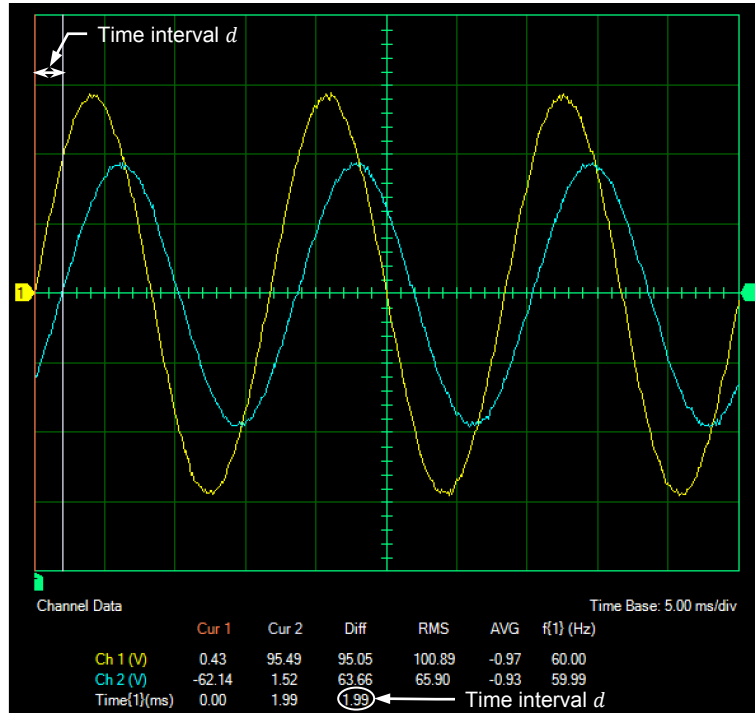
Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Time interval  $d = 1.99$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 60 Hz.

16. Using Equation (1-11), calculate the phase shift between the source voltage  $E_S$  and the resistor voltage  $E_R$ . Consider the source voltage waveform as the reference.

Phase shift = \_\_\_\_\_ °

50 Hz: Phase shift =  $\frac{d}{T} \cdot 360^\circ = \frac{2.53 \text{ ms}}{20.1 \text{ ms}} \cdot 360^\circ = 45.3^\circ$

Phase shift = 45.3° lagging or -45.3°

60 Hz: Phase shift =  $\frac{d}{T} \cdot 360^\circ = \frac{1.99 \text{ ms}}{16.7 \text{ ms}} \cdot 360^\circ = 42.9^\circ$

Phase shift = 42.9° lagging or -42.9°

17. Is the resistor voltage  $E_R$  leading or lagging the source voltage  $E_S$ ?

The resistor voltage  $E_R$  is lagging the source voltage  $E_S$ .

18. In LVDAC-EMS, open the Phasor Analyzer and display the source voltage  $E_S$  (input E1) and resistor voltage  $E_R$  (input E2). Set the Reference Phasor parameter to E1. Measure the phase angles  $\theta_{ES}$  and  $\theta_{ER}$  of the voltage phasors.

Phase angle  $\theta_{ES} = \underline{\hspace{2cm}}^\circ$

Phase angle  $\theta_{ER} = \underline{\hspace{2cm}}^\circ$

From these values, calculate the phase shift between the phasors of the source voltage  $E_S$  and resistor voltage  $E_R$ , using the source voltage phasor as the reference.

Phase shift =  $\underline{\hspace{2cm}}^\circ$

50 Hz: Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = -43.8^\circ$

Phase shift =  $43.8^\circ$  lagging or  $-43.8^\circ$

60 Hz: Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = -42.3^\circ$

Phase shift =  $42.3^\circ$  lagging or  $-42.3^\circ$

19. Compare the phase shift you determined from the voltage sine waves to the phase shift you measured from the corresponding voltage phasors. Are both values close to each other?

Yes     No

Yes

### Measuring the phase shift between two voltage sine waves in a resistor-capacitor (RC) circuit

*In this section, you will replace the inductor used in the previous section by a capacitor. Using the Oscilloscope, you will determine the phase shift between the two voltage sine waves. Then, using the Phasor Analyzer, you will measure the phase shift between the source voltage phasor and the resistor voltage phasor and compare it to the phase shift you determined from the voltage waveforms.*



*As you will see later, due to the presence of a capacitor in the circuit, the circuit current leads the source voltage. As a result, the resistor voltage  $E_R$  is out of phase with respect to the source voltage  $E_S$ .*

20. In the Four-Quadrant Dynamometer/Power Supply window, disable the ac power source.



21. Modify the circuit so that it is as shown in Figure 1-16 (replace the inductor by a capacitor). This circuit contains a resistor  $R$  and a capacitor  $C$ . Capacitors are studied in the next unit of this manual.

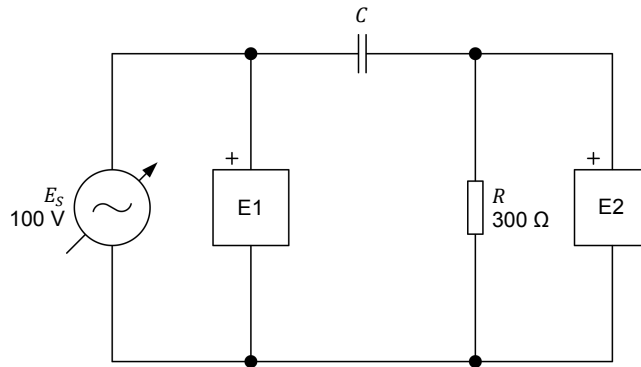


Figure 1-16. AC circuit with a resistor and a capacitor.

The value of capacitor  $C$  in the circuit of Figure 1-16 is referred to as the capacitance and is expressed in microfarads ( $\mu\text{F}$ ). The capacitance value to be used depends on the frequency of the ac power source as is indicated in Table 1-3.



As indicated in Appendix A, use the [Capacitive Load](#) module to obtain the required capacitance when the ac power network frequency is 60 Hz. Use the [Inductive and Capacitive Loads](#) module to obtain the required capacitance when the ac power network frequency is 50 Hz.

Table 1-3. Capacitance values for 50 and 60 Hz frequencies.

Power source frequency (Hz)	Capacitance ( $\mu\text{F}$ )
50	5.3
60	4.4

Make the necessary switch settings on the [Resistive Load](#) in order to obtain the resistance values required.



Appendix C of this manual lists the switch settings to implement on the [Resistive Load](#) in order to obtain various resistance values.

Make the necessary connections and switch settings on the [Capacitive Load](#) (or on the [Inductive and Capacitive Loads](#)) in order to obtain the capacitance value required.



If necessary, ask your instructor to assist you to obtain the capacitance value required.

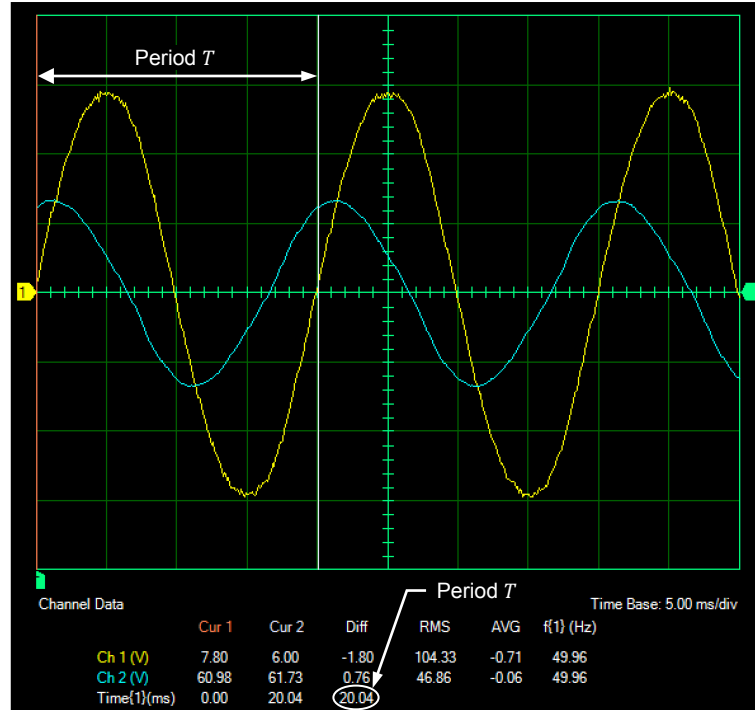
22. In the [Four-Quadrant Dynamometer/Power Supply](#) window, enable the ac power source. Readjust the value of the [Voltage \( \$V\$  at no load\)](#) parameter, if necessary, so that the ac power source voltage  $E_s$  (indicated by meter  $E1$  in the [Metering](#) window) is equal to 100 V.

23. Measure the period  $T$  of the source voltage using the Oscilloscope then record the value below.

Period  $T =$  \_\_\_\_\_ ms

50 Hz: Period  $T = 20.04$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising

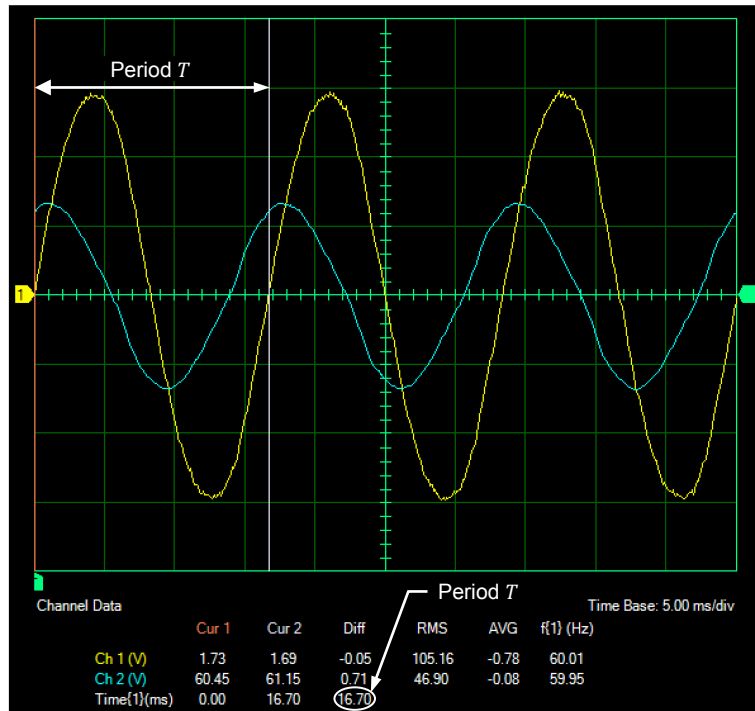


Period  $T$  of the waveform of the source voltage  $E_S$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.70$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



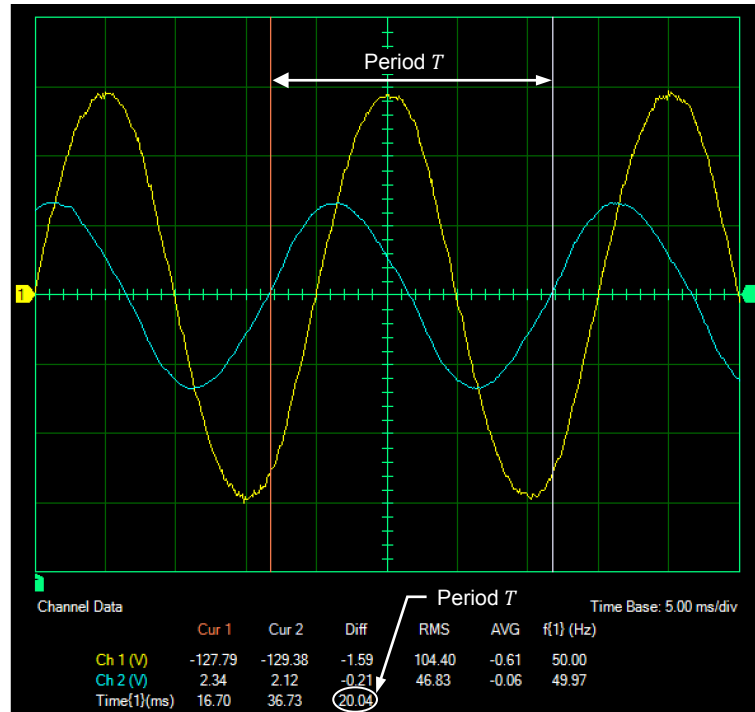
Period  $T$  of the waveform of the source voltage  $E_S$  at a frequency of 60 Hz.

24. Measure the period  $T$  of the resistor voltage  $E_R$  using the Oscilloscope then record the value below.

Period  $T =$  \_\_\_\_\_ ms

50 Hz: Period  $T = 20.04$  ms. The results are shown in the following figure.

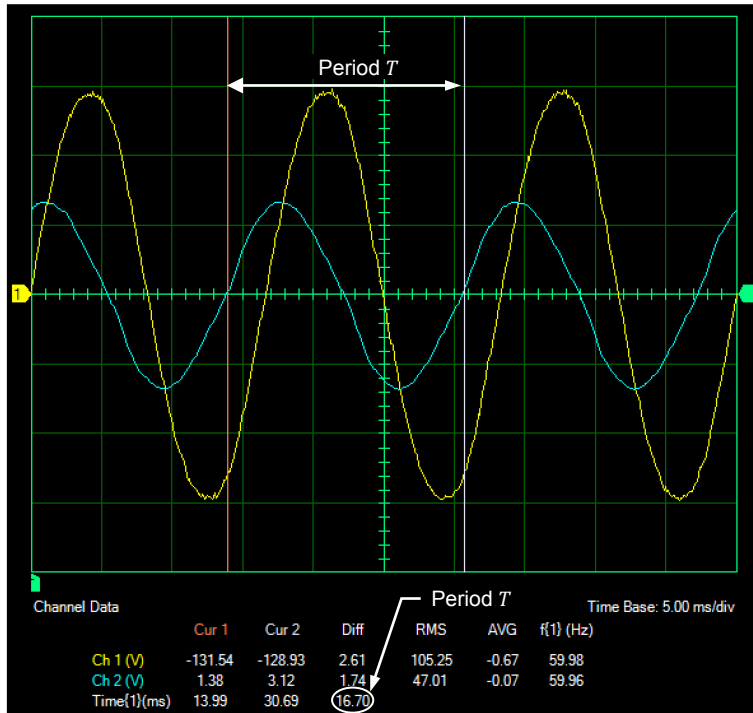
Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Period  $T = 16.70$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Period  $T$  of the waveform of the resistor voltage  $E_R$  at a frequency of 60 Hz.

25. Compare the period  $T$  of the resistor voltage  $E_R$  measured in the previous step with the period  $T$  of the source voltage  $E_S$  recorded in step 23. Are the values close to each other?

Yes     No

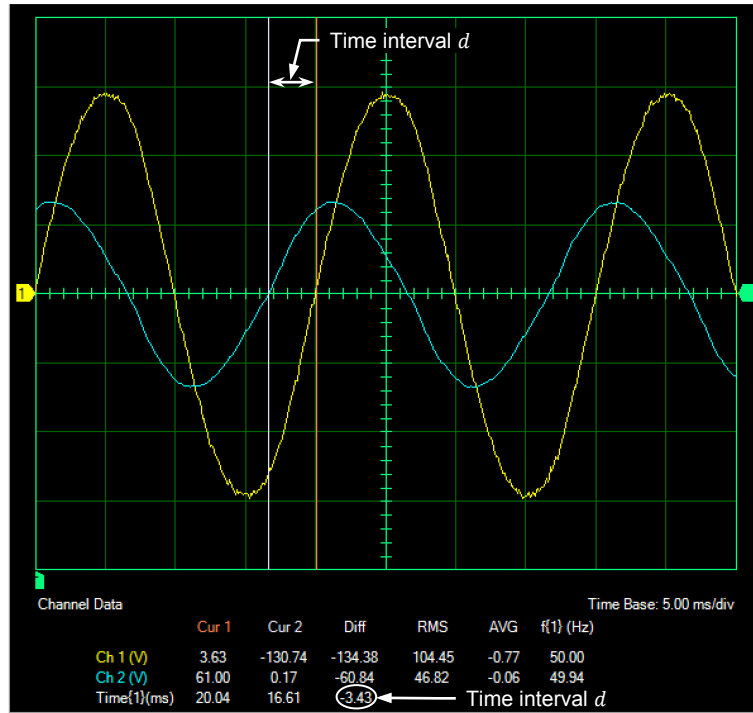
Yes

26. Measure the time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$ .

Time interval  $d = \underline{\hspace{2cm}}$  ms

50 Hz: Time interval  $d = 3.43$  ms. The results are shown in the following figure.

Oscilloscope Settings  
 Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising

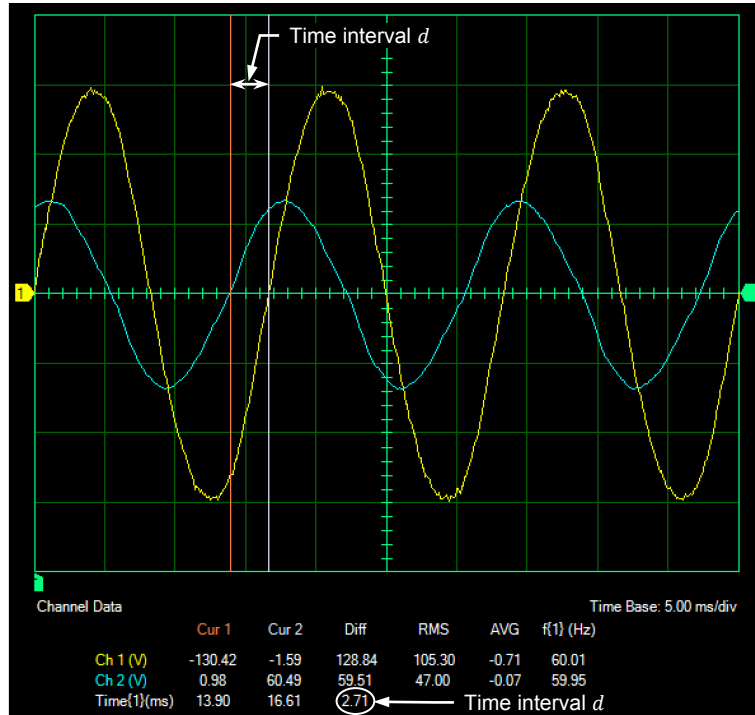


Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 50 Hz.

60 Hz: Time interval  $d = 2.71$  ms. The results are shown in the following figure.

Oscilloscope Settings

Channel-1 Input ..... E1  
 Channel-1 Scale ..... 50 V/div  
 Channel-1 Coupling ..... DC  
 Channel-2 Input ..... E2  
 Channel-2 Scale ..... 50 V/div  
 Channel-2 Coupling ..... DC  
 Display Filtering ..... On  
 Show Cursors ..... Vertical  
 Trigger Type ..... Software  
 Time Base ..... 5 ms/div  
 Trigger Source ..... Ch1  
 Trigger Level ..... 0  
 Trigger Slope ..... Rising



Time interval  $d$  between the waveforms of the source voltage  $E_S$  and resistor voltage  $E_R$  at a frequency of 60 Hz.

27. Using Equation (1-11), calculate the phase shift between the source voltage  $E_S$  and the resistor voltage  $E_R$ . Consider the source voltage waveform as the reference.

Phase shift = \_\_\_\_\_ °

50 Hz: Phase shift =  $\frac{d}{T} \cdot 360^\circ = \frac{3.43 \text{ ms}}{20.0 \text{ ms}} \cdot 360^\circ = 61.7^\circ$

Phase shift = 61.7° leading or 61.7°

60 Hz: Phase shift =  $\frac{d}{T} \cdot 360^\circ = \frac{2.71 \text{ ms}}{16.7 \text{ ms}} \cdot 360^\circ = 58.4^\circ$

Phase shift = 58.4° leading or 58.4°

28. Is the resistor voltage  $E_R$  leading or lagging the source voltage  $E_S$ ?

The resistor voltage  $E_R$  is leading the source voltage  $E_S$ .

29. In the **Phasor Analyzer**, measure the phase angles  $\theta_{ES}$  and  $\theta_{ER}$  of the voltage phasors.

Phase angle  $\theta_{ES} = \underline{\hspace{2cm}}$  °

Phase angle  $\theta_{ER} = \underline{\hspace{2cm}}$  °

From these values, calculate the phase shift between the phasors of the source voltage  $E_S$  and resistor voltage  $E_R$ , using the source voltage phasor as the reference.

Phase shift =  $\underline{\hspace{2cm}}$  °

50 Hz: Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = 63.4^\circ$

Phase shift =  $63.4^\circ$

60 Hz: Phase angle  $\theta_{ES} = 0^\circ$

Phase angle  $\theta_{ER} = 63.5^\circ$

Phase shift =  $63.5^\circ$

30. Compare the phase shift you determined from the voltage sine waves to the phase shift you measured from the corresponding voltage phasors. Are both values close to each other?

Yes     No

Yes

31. In the **Four-Quadrant Dynamometer/Power Supply** window, disable the ac power source.
32. Close **LVDAC-EMS**, then turn off all the equipment. Disconnect all leads and return them to their storage location.

## CONCLUSION

In this exercise, you saw how the phase angle modifies the value of a sine wave at time  $t = 0$ , and thus, the position in time of the sine wave. You observed the effects of positive and negative phase angles on the relative position in time of a sine wave. You were introduced to the notion of phase shift. You learned how to calculate and measure the phase shift between two sine waves and to differentiate between a lagging and a leading phase shift.



**REVIEW QUESTIONS**

1. What is the effect of the phase angle on the graphical representation of a sine wave?

The phase angle determines the value of a sine wave when  $t = 0$  s, and thus, the position in time of the sine wave.

2. A sine wave has a phase angle  $\theta$  of  $72^\circ$ . Will this sine wave reach its maximum value before, after or at the same time as a second waveform having a phase angle  $\theta$  of  $-18^\circ$ ?

The sine wave with the phase angle  $\theta$  of  $72^\circ$  will reach its maximum value before the sine wave having a phase angle  $\theta$  of  $-18^\circ$ .

3. Given the following two sine wave equations:

$$E(t) = 8 \sin 20t + 78^\circ$$

$$E(t) = 40 \sin 20t + 43^\circ$$

Calculate the phase shift between these two sine waves, considering the first sine wave as the reference. Indicate also whether the second sine wave is lagging or leading the reference sine wave.

$$\text{Phase shift} = 43^\circ - 78^\circ = -35^\circ$$

The second sine wave is lagging the reference (first) sine wave.

4. When calculating the phase shift between two sine waves, which of the following parameters do the two sine waves need to have in common: phase angle, amplitude, frequency, or period? Why?

To calculate the phase shift between two sine waves, it is necessary for them to have the same frequency and period. Amplitude and phase angle have no effect on the duration of the sine wave cycle.

5. Consider two sine waves with the same frequency. They both have a period  $T$  of 50 ms. The second sine wave reaches its maximum positive value 8 ms after the first. Calculate the phase shift between the two sine waves, considering the first one as the reference.

$$\text{Phase shift} = \frac{d}{T} \cdot 360^\circ = \frac{8 \text{ ms}}{50 \text{ ms}} \cdot 360^\circ = 57.6^\circ$$

Since the second sine wave is lagging the reference, the value of the phase shift is negative, thus:

$$\text{Phase shift} = -57.6^\circ \text{ or } 57.6^\circ \text{ lagging}$$



# Bibliography

Boylestad, Robert L., *Introductory Circuit Analysis*, 11th ed., Upper Saddle River: Prentice Hall, 2006, ISBN 978-0131730441.

Wildi, Theodore, *Electrical Machines, Drives, and Power Systems*, 6th ed., Upper Saddle River: Prentice Hall, 2005, ISBN 978-0131776913.